### The Market for Lemmas

by Philip R.P. Coelho and James E. McClure

It is frustrating but nevertheless true that, where mathematics is most likely to be useful, the theory is least likely to be valid, while, where the theory is most likely to be true, complex deduction is generally not needed.

Donald F. Gordon (1955, p. 160)

..., from a certain love of art and contrivance, we sometimes seem to value the means more than the end, and to be eager to promote the happiness of [people], rather from a view to perfect and improve a certain beautiful and orderly system, than from any immediate sense or feeling of what [people] either suffer or enjoy.

Adam Smith (1759; 2007, Part IV, paragraph IV.I.11)

Resources are scarce relative to human wants; economists weave this perspective into theories whose implications are assessed econometrically against data generated *independently* of the theory. Theories that do not provide testable propositions at reasonable costs are usually disregarded. The evidentiary warehouse that allows theories to be tested is observational reality. The potential to refute separates theories from other statements. Evidence and data are not absolutes; they do not simply exist waiting to be recognized by investigators; they have to be extracted and manipulated for the purposes at hand. Evidence is judgmental; it is mined from a myriad of available data and facts, and it is evaluated by an increasingly powerful variety of techniques. In distinguishing between dogmas from causal explanations, evidence beyond the confines of a theory gives researchers the ability to judge whether the theory is consistent with observational reality.

The ability to formalize refutable statements and find replicable evidence that is either consistent or inconsistent with them is operationalism. Statements that cannot be challenged are what Wolfgang Pauli famously dismissed as "not even wrong." Statements that have been refuted educate people and provide information on what not to do; non-operational statements

lack this saving grace. Econometrics provides the statistical and mathematical mechanisms that allow theories to be assessed.

All assessments require the specification of appropriate background conditions because no theory is universally applicable regardless of time, place, and other circumstances. What is appropriate is a judgmental (normative) exercise that depends upon the particular application of the theory, the costs of establishing background conditions and data acquisition. After establishing the conditions, judgments about whether the collected observations are inconsistent with the hypotheses can be considered.<sup>2</sup> Operational statements are made about observational reality and are assessed empirically.<sup>3</sup> While such statements cannot be proved *true*, such statements must be capable of being shown as either consistent, or inconsistent with observational reality.<sup>4</sup> Operational statements that are inconsistent with the available evidence are discarded.<sup>5</sup> Empirical tests of operational statements: 1) identify theories whose predictions are inconsistent with experience; and 2) demark the set(s) of experiences within which theories apply.<sup>6</sup>

## I. Employing Mathematics: Alternative Usages

Contrast the use of mathematics in economics with its use in econometrics. Major journals in economics are increasingly publishing theories of economic behaviors that are neither concerned with the generation of operational statements nor any evidentiary assessment of their utility. They typically emphasize the manipulations of sophisticated mathematical systems and are generally devoted to two tasks: the first is establishing a story consistent with a set of *stylized facts* and formulating it into a mathematical model that can be tractably handled by the tools available to the authors. The second task is deriving proofs of the mathematical consistency of

the model. These *proofs* are not evidence supporting or rejecting hypotheses, they rely upon neither data, nor history but simply test the internal consistency of the mathematics. The term *proof* here means only that there were no obvious inconsistencies in the mathematical model. Frequently these articles mention operationalism with statements (usually in the concluding segments) that the "theories" presented *may* lead to operational propositions in "future research." Bayesian priors suggest that the probability of a subsequent article appearing with refuting data, or even any data, is approaching zero. Evidence exists that mathematically complex articles are less likely to contain empirics and less likely to generate empirical research than less mathematically complex articles. Another reason that complex articles rarely generate operational research is because their many contrary-to-fact assumptions make testing the models extraordinarily difficult, consequently creating defensible econometric specifications and test conditions becomes quixotic at best. 11

In contrast to analyzing the internal consistency of economic theories, the mathematical proofs in econometric work that ensure internal consistency of tools and techniques employed in statistical pursuits are not affected by the passage of time. Observational reality (the subject of economic theories) changes over time, in contrast econometric tests (the subject of econometric theory) are unaltered by the passage of time. A century from now the calculation of a Chisquared statistic will require the same mathematical steps that are used today. The economic relationships between the variables in the Chi-squared contingency tables may/will be altered by time, but the test itself is a mathematical abstraction that has no counterpart in physical or behavioral reality and is immutable to the ravages of time.

Corresponding to economic theories there is an economic reality that the theory must

explain or it is not an *economic* theory. Only operational statements (those that can be assessed against data generated by real-world observations) are valid theories in explaining observational reality. Because the realms of mathematics and observational reality are not the same, special care has to be used in applying mathematical tools and conventions to insure operationalism. In a famous passage Alfred Marshall (1920; 1964) both condemned and praised complex mathematical reasoning in economics.

It is obvious that there is no room in economics for long trains of deductive reasoning; no economist, not even Ricardo, attempted them. It may indeed appear at first sight that the contrary is suggested by the frequent use of mathematical formulae in economic studies. But on investigation it will be found that this suggestion is illusory, except perhaps when a pure mathematician uses economic hypotheses for the purpose of mathematical diversions; for then his concern is to show the potentialities of mathematical methods on the supposition that material appropriate to their use had been supplied by economic study. He takes no technical responsibility for the material, and is often unaware how inadequate the material is to bear the strains of his powerful machinery. But a training in mathematics is helpful by giving command over a marvelously [sic] terse and exact language for expressing clearly some general relations and some short processes of economic reasoning; which can indeed be expressed in ordinary language, but not with equal sharpness of outline. And, what is of far greater importance, experience in handling physical problems by mathematical methods gives a grasp, that cannot be obtained equally well in any other way, of the mutual interaction of economic changes. (p. 644)

Paul Samuelson notes that both Alfred Marshall and John Stuart Mill spoke "of the dangers involved in *long* chains of logical reasoning;" and he explains that:

Marshall treated such chains as if their truth content was subject to radioactive decay and leakage-at the end of n propositions only half the truth was left, at the end of a chain of 2n propositions, only half of half the truth remained, and so forth in a geometric multiplier series converging to zero truth. (1952, p. 57)

Building upon this theme, Donald F. Gordon hypothesized that the relationship between "operationalism" and mathematical sophistication was negative. Gordon explained the operational difficulty created by having many mathematical functions via an example of a theory relating three

distinct variables x, y, and z:

Again, the relationship between x and y may be stable long enough for a shift along that function but not stable long enough for a shift along that function plus a subsequent shift along another [z].(p. 155)

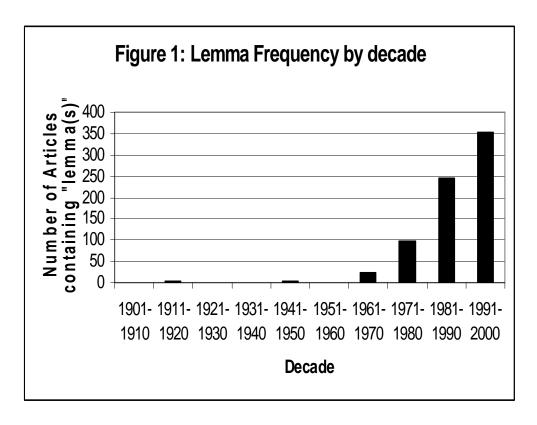
Problems with the theory arise if the *ceteris paribus* assumption breaks down as it would if the relationship between y and z does not occur instantaneously.<sup>12</sup> Arguing that *economic* phenomena are time-dependent, Gordon hypothesized that as the length of a mathematical chain increased in an *economic theory*, it would become increasingly likely that the passage of time would compromise (in unpredictable ways) the *assumed* stability of the chain's linkages. The timelessness implicit in multiple mathematical linkages was seen by Gordon as an obstacle to operationalizing complex mathematical theories about economic phenomena.<sup>13</sup>

## II. Some Evidence on Complexity and Operationalism

The mathematical proofs used in economic theory to test for internal consistency differ from econometric work because they are not designed to be used to test proposition or to be tested. The mathematics used in many "pure" theory papers in economics are more like ornamental artifacts rather than a necessary step to an underlying goal (operationalism). The theorems, lemmas and other intermediate steps that populate in the mathematical proofs in pure theory appear to be there for normative reasons because they are generally non-operational.

Lemmas are the intermediate steps in proofs that are sufficiently complex and/or lengthy that delineation of intermediate steps (*Lemma 1, Lemma 2...et cetera*) are deemed crucial for comprehension. Whether mathematical complexity is required to advance the argument depends upon the usage, context, and purpose; for example, the creation of a novel econometric technique demands a complete investigation of its mathematical implications and internal consistency. We

are explicitly differentiating between economics and econometrics in the employment of highly sophisticated mathematical techniques. In "pure [economic] theory", whenever the use of advanced mathematics becomes an end unto itself, the theory is self-referential. We use the presence of the word "lemma(s)" as an indicator of mathematically complex modeling in articles in economics and econometrics. The term "lemma(s)" has become increasingly frequent in the journal literature; Figure 1 illustrates this for four prominent journals in economics. The vertical axis of Figure 1 represents the numbers of articles found per decade in a full-text search of the JSTOR data base that contain either the term *lemma* or *lemmas* in the *AER*, *EJ*, *JPE* and *QJE*. <sup>15</sup>



The appearance of the word "lemmas" was rare in the first six decades of the twentieth century, but during the last four decades it became increasingly frequent.<sup>16</sup>.

Table 1 provides evidence bearing upon the proposition that mathematical complexity

of Economic Theory that contained 5 or more lemmas. The articles are listed by author(s); the other columns list: a) how many lemmas each article had, b) how many citations each article had up to the present, c) how many of the citing articles had empirical data, d) how many of the citing articles empirically tested a proposition of the original article, and e) how many citing articles refuted a proposition of the originating article. <sup>19</sup>

TABLE 1: Characteristics of articles containing 5 or more Lemmas in the Journal of Economic Theory in 1980 [by author(s) and characteristics]						
Author(s)	Number of Lemmas in Article	Number of Cites to the Author(s)				
		Total	Containing Empirics	Attempting Direct Empirical Assessment	Empirical Assesments that Accept or Reject	
Kalai & Ritz	6	9	0	0	0	
Cohen	6	0	0	0	0	
Green	5	41	3	1	0	
Makowski	8	17	0	0	0	
Dubey	6	15	1	0	0	
Gaines	7	2	0	0	0	
Krass	5	2	1	1	0	
Rubenstein	6	13	0	0	0	
Flaherty	6	21	2	0	0	
Kleinberg	5	6	0	0	0	
Balasko & Shell	13	103	2	0	0	
Littlechild & Owen	6	8	0	0	0	
TOTALS	79	237	9	2	0	

There were 12 articles in 1980 that had five or more lemma generated 237 citations to them in the following (approximately) quarter century. Nine of the 237 citing articles contained empirical data, two had empirical data that had something to do with the propositions of the original article, and none had a definitive test leading to an acceptance or rejection of a proposition of the original article. In short, the originating articles have to date defined no

operational propositions.<sup>20</sup>

# III. Complexity in Econometrics vs. Economics: What do "Best Practices" Suggest?

The data in Table 1 are not definitive; whether the hypothesis that mathematical complexity and operationalism are negatively related in economic theories requires more investigations. If the hypothesis is incorrect, then one could reason that mathematically complex articles would be among the most important and cited articles. Kim, Morse, and Zingales (2006) have compiled a comprehensive list of articles published between 1970 and 2002 in 41 prominent journals in economics (and econometrics) that generated at least 500 citations to them as of June 2006. We take this as a list of articles that illustrates the best practice and have had the greatest impact. Using the data from Kim, *et al.*(Table 2, p.15) we took all the articles that were published in four top general interest journals (*AER*, *EJ*, *JPE*, and *QJE*) that had 500 or more citations and examined the frequency of lemmas in each of the articles. Table 2 summarizes our findings:

Table 2: Lemma Usage in the Most Widely Cited Articles in Top General Interest Journals in Economics					
Journal	Total Number of articles*	Number of articles that created at least one			
		lemma			
AER	19	0			
EJ	4	0			
JPE	26	0			
QJE	11	1			
TOTALS	60	1			

<sup>\*</sup>Data in this column were extracted from Table 2 of: Kim, Morse, and Zingales (2006, p. 15)

Of the sixty articles in AER, EJ, JPE, and QJE that had been cited more than 500 times, only one

article contained an author written lemma(s).<sup>21</sup> These data are inconsistent with using mathematical complexity as an indicator of "best practice" and/or greatest impact in the articles appearing in these journals.

The data in Table 3 do not address the Gordon hypothesis relating the negative relationship between mathematical complexity and operationalism in economics. Gordon's hypothesis pertains to theories about economic phenomena; his hypothesis **does not** pertain to the formulation of econometric tools and techniques that are formulated to test propositions about economic phenomena. In the derivation of such a tool, Gordon's hypothesis suggests no reason to expect there to be a negative relationship between mathematical complexity and the usefulness of the tool. From the list of Kim, Morse, and Zingales, we extracted those articles published in *Econometrica* and *The Journal of the American Statistical Association* that were cited 500 or more times. We examined each articles to see how frequently they contained at least one lemma (our proxy for mathematical complexity). Table 3 summarizes our findings:

Table 3: Lemma Usage in the Most Widely Cited Articles in Statistical/Econometrics Journals					
Journal	Total Number of articles*	Number of articles that created at least one lemma			
Econometrica	31	14			
Journal of the American Statistical Association(JASA)	6	4			
TOTALS	37	18			

<sup>\*</sup> Source of data in this column: Kim, Morse, Zingales (2006; Table 2, p. 15)

Of the 37 articles in *Econometrica* and *The Journal of the American Statistical Association* that have been cited more than 500 times, 18 created lemmas.

The difference between the results of this table and those found in Table 2, where only 1

in 60 of the articles contained author created lemmas is stark.<sup>22</sup> A statistical test of this difference is made based using Table 4 below:

Table 4: Contingency Table (Most Widely Cited Articles in Alternative Types						
of Journals by Lemma usage)						
Journal Type	Articles with zero author	Articles with one or more				
	created lemmas	author created lemmas				
General Interest Journals	59	1				
(AER, EJ, JPE, QJE)						
Econometrics/Statistical	19	18				
(Econometrica, JASA)						

The  $\chi^2$  statistic for this contingency table is 32.05; this is significant 1% level. This evidence leads to a rejection of the hypothesis that lemma usage in the most cited articles in economics is as frequent as in the most-cited articles in econometrics/statistical journals.

There is evidence that mathematical complexity suppresses operationalism in economics. In econometrics there is no evidence suggesting a negative relationship between operationalism and complexity. Kim, Morse and Zingales (Table 2, p. 15) have listed "what matters" in most in economics, displaying by authors and title the articles in descending order by the number of citations. The most widely cited are articles that supply econometric tests and techniques for the manipulation of data. These articles are widely cited because the tools they supply are useful for examining data in citing articles that are operationalizing theories.

# **VI.** Concluding Remarks

Mathematics has benefited economics because it is a useful tool for: thinking systematically and concisely, verifying the internal consistency of arguments, exposing the similarities and differences between alternative theories, and for developing successful statistical/econometric tools and techniques. This paper has been about the displacement of

operationalism as the core pursuit of economics by the pursuit of mathematical elegance and generality. Non-operational models of economic phenomena are worse than wrong in a scientific sense because they draw resources away from the creation and examination of operational propositions and do not provide any information or hypotheses about observable reality.

Alfred Marshall (p. 1) stated that economics is: "... a study of mankind in the ordinary business of life . . . ." This is in contrast to the mathematical ideal of generality, elegance, and "pure" theory unblemished by the pursuit of material considerations. If we are dealing with the "ordinary business of life" we are unlikely to encounter either absolute "Truth" or elegance that is sought by purists. In the Marshallian tradition the best we can hope for are conditional statements that are dependent upon time and a host of other circumstances; here the use of mathematics will be tempered by measurements, operationalism, experience, history, and all the nuances that are relevant to the purpose(s) at hand.

During the last century, economists have discussed the implications of mathematically complexity in economic theory. In 1920, Alfred Marshall stated that it was "obvious" that there was "...no room in economics for long trains of deductive reasoning." What was *obvious* to Marshall was not *obvious* to the economics profession *writ large*. In the mid-twentieth century the increasing mathematical complexity of economics led Donald Gordon (1955, p. 161) to speculate that concerns for operationalism in economics implied that "the practice of proliferating and manipulating functions has gone to somewhat incautious limits."

The evidence presented here indicates that mathematical complexity in economics has expanded exponentially beyond the levels that Gordon decried as "incautious." Mathematical

complexity has commanded more resources in economics, yet the additional complexity has generated few, if any, operational propositions about the world in which we live and no refutable propositions about the theories the complexity spawned.<sup>23</sup> The concern for operationalism - measurement, empiricism, statistical testing, and/or history - is the focus of an economics discipline that attempts to explain phenomena that have counterparts in observational reality. Complex mathematics that does not further these goals has to be justified by non-utilitarian norms.

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#### **Endnotes:**

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¹Independently, means theories are not used to construct the data. An example of data that is not independent in *The Historical Statistics of the United States* is on farm capital; the data were constructed by the US Department of Agriculture using depreciation models derived from economic theory. It is not surprising to see good correlations between these *data* and models. See Edward Leamer (1978) other difficulties arising when hypotheses are tuned to fit the data.

¹This is a relatively "Popperian" (1934/1959) view on operationalism and its relation to science.

³We are not operational absolutists, the data do no confess, they are interpreted; we elaborate this in the next section. Essentially we argue that economists make *reasonable* searches for knowledge in the world that surrounds them. In this we are in close accord with Donald N.

McCloskey's (1983) appeal for reason and restraint in argumentation.

<sup>&</sup>lt;sup>4</sup> Farley Grubb's (2001) identifies two forms of ideas: useful ideas and meaningful ideas. In his categorization "useful ideas are assessed by the patterns discovered within our sensory universe." On the other hand, "meaningful ideas are assessed by rhetoric." (p. 6) We equate his term "sensory universe" with our "observational reality." This puts our usage of the term "operationalism" under the rubric of Grubb's "useful ideas." As we see it an operational proposition is assess by patterns "discovered within our sensory universe."

<sup>&</sup>lt;sup>5</sup>According to Donald F. Gordon (p. 150) an operational proposition is "one that implies or states an operation that could, in principle, be performed, the result of which tests the proposition." Consistent with our view he continues (on the same page) that: "For a proposition to be testable by some physical operation, it must predict that certain observations will *not* occur, so that, if they do, the proposition is refuted." (his emphasis)

<sup>&</sup>lt;sup>6</sup>For example, the theory of the acceleration of objects in a vacuum is of little use in predicting the speed of a falling feather in a gale, but is useful for bowling balls dropped short distances

close to the earth's surface.

13 Wassilly Leontief (1971, 1-2) echoed Gordon's concerns about the timelessness implicit in mathematics: "Uncritical enthusiasm for mathematical formulation tends often to conceal the ephemeral substantive content of the argument behind the formidable front of algebraic signs."

14 There are prominent critics of the current-day usage of sophisticated mathematics in economics. Gordon Tullock (1993, p. 26) argued that pure research using mathematic in economics may provide a shortcut to "... an improved type of applied research..." yet he was (at best) skeptical of the claims for mathematical research in economics, but admitted that "... present data on the subject is weak enough so I cannot be certain...." McCloskey (1994, p. 131) is critical of economists who embrace formalism because they: 1) "have adopted the intellectual values of the math department" which are "proof driven," and devoted to "complete accuracy"; and 2) have eschewed "the values of the departments of physics or electrical engineering or biochemistry" that are happy to use "approximations and simulations and empirically relevant parameters, not existence theorems" (p. 134) when they provide sufficient accuracy for the operations at hand. Farley Grubb, primarily an economic historian, calls these

<sup>&</sup>lt;sup>7</sup> Coelho and McClure (2005) present evidence that in the *AER*, *EJ*, *JPE*, and *QJE* between 1963 and 1996: "Mathematically complex articles were less operational and were less likely to be cited in articles containing operational statements." (p. 565)

<sup>&</sup>lt;sup>9</sup> Gödel's Theorem on the inadequacy of mathematical systems to determine the truth or falsity of all mathematical statements is simply ignored.

<sup>&</sup>lt;sup>10</sup> See Coelho and McClure (2005).

<sup>&</sup>lt;sup>11</sup> See Daniel M Hausman (1989, pp. 120-121) on "unrealistic" assumptions, and on "tractability" see Frank Hindriks (2005).

<sup>&</sup>lt;sup>12</sup> In this case the syllogism is not valid.

articles "nonscientific":

A casual reading of journals of economic theory will reveal many discourses overwhelmed by pedantry, that is, mathematical formalization for its own sake (a failure to realize that translation is not analysis), exercises that generate no practical empirically testable hypotheses, and interpretations of meaning that over-step the underlying scientific precepts of economic methodology. (p. 10)

Robert Sugden (2000), a mathematical economist, expressed the following sentiments about theorists who are disinterested in real-world credibility:

I have no fellow-feeling with those economic theorists who, off the record at seminars and conferences, admit that they are only playing a game with other theorists. If their models are not intended seriously, I want to say (and do say when I feel sufficiently combative), why do they expect me to spend my time listening to their expositions? Count me out of the game. (p. 1)

<sup>15</sup> The year 2000 was chosen as the last year of consideration because JSTOR does not provide data for all of the four journals considered beyond that year. The first appearance of lemma(s) in any of these journals was in Edgeworth's (1910) article in *EJ*. Although the graph shows the trend clearly, its scaling makes it difficult to gauge the numbers for early decades; we list the numeric results here: 1900-1910 (*one article*); 1911-1920 (*two articles*); 1921-1930 (*zero*); 1931-1940 (*zero*); 1941-1950 (*two*); 1951-1960 (*one*); 1961-1970 (22); 1971-1980 (98); 1981-1990 (245); 1990-2000 (353).

<sup>&</sup>lt;sup>16</sup> More inclusive measures of "mathematical complexity" could have been presented, but simply looking at lemma(s) is "good enough" to illustrate the trend towards publication of articles of increasing mathematical complexity.

The *ISI Web of Science* was used to identify citations. Our search of this database occurred during the first two weeks of June of 2006, and was from 1980 to the present. After citations were identified, each citing article was individually inspected to see whether it: a) contained empirics, b) attempted a direct assessment of any of the author(s) theoretical propositions, and c) contained empirical assessments that accepted or rejected any proposition of the author(s).

Citing articles containing only casual empiricism (stylized facts) were not counted as containing empirics, nor did the presence of self-referential simulations qualify them as containing empirics.

However, citing articles containing data from surveys and/or experiments did qualify them as containing empirics.

For the originating (1980) articles listed in Table 1, the average number of lemmas per article is (79/12) or 6.58. For comparison purposes we counted the numbers of articles in *JET* in 2005 having 5 or more lemmas (there were 21 such article), and we counted the numbers of lemmas in these articles (there were 165 lemmas). In the 2005 set of articles the lemmas per article was 7.86. Comparing *JET* in 1980 to 2005, there has been an increase in the number of articles using 5+ lemmas and the mean number of lemmas in these articles increased.

<sup>&</sup>lt;sup>21</sup> Cho and Kreps (1987) used two lemmas in their publication in the *QJE*.

<sup>&</sup>lt;sup>22</sup> Of course, not all of the articles in these journals involve the development of statistical tests. To be sure that the finding above did not overstate the difference, we inspected all the above articles to see if they concerned the development of statistical tests. While all of the articles in the *Journal of the American Statistical Association* concerned the development of statistical test, only 21 of those in *Econometrica* did so; still, of these 21 articles, 10 contained author-created lemmas. Revising the totals in Table 3 to reflect only articles concerned with development of statistical tests, the contrast with Table 2 would only become more pronounced in percentage terms: There would be 14 out of 27 in a revised Table 3, versus, again, 1 out of 60 in Table 2.

<sup>23</sup> In explaining *why* mathematical complexity spreads, Gordon Tullock (2005, p. 47) reasoned that it spreads in fields where opportunities for original research are limited relative to the number of people in the field. "One symptom of the existence of this condition is the development of very complex methods. Calculus will be used where simple arithmetic would do, and topology will be introduced in place of plane geometry. In many fields of social science

these symptoms have appeared."