Firm market value and production technology ☆

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ABSTRACT

This paper estimates the production technology of the U.S. computer industry using firm market value to control for the correlation between inputs and unobservable productivity shocks. We show that firm market value can serve as a proxy for unobservable productivity shocks. We also show that firm market value is robust as a proxy when firm faces uncertainties and capital market imperfections. Empirical results suggest that our firm market value proxy works well for the computer industry.

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1. Introduction

Empirical work depends on observables. Production technology is affected by a mix of observable factors, such as expenditures on capital and labor, and unobserved productivity shocks, such as how capably managers deploy these resources. Consequently, economists who study technology must isolate echoes of unobserved productivity shock in available data.

We use firm market value to control for such unobservable characteristics in an analysis of the U.S. computer industry. We build on Olley and Pakes (1996) and Levinsohn and Petrin (2003). In their model, firms have heterogeneous marginal products of factor inputs. Economists cannot observe this heterogeneity directly, but by inverting the relationship between investment or intermediate inputs and firm heterogeneity, they can control for it. We show that firm market value provides an additional means to distinguish among heterogenous firms.

We apply our method to the U.S. computer industry. The computer industry is characterized by rapid change, and thus provides an appropriate subject of study for estimating technologies that permit time-varying firm effects. We use a unique data set from the merger of Compustat data and establishment-level information based on Dun and Bradstreet's archives.

This paper is organized as follows. In Section 2, we review methods of estimating production functions. In Section 3, we prove that firm market value, like investment, can be a proxy for unobservable productivity shocks. We describe our data set in Section 4. In Section 5, we apply our proxy to the U.S. computer industry, and compare the results of the value proxy to estimates generated using fixed effects and the investment proxy. Section 6 discusses the limitations of our approach and concludes.

2. Estimating production functions

A wide variety of information affects firms' production decisions. Some information, such as change in inventory, can be summarized by a number. Other characteristics, such as the hiring of a new manager or the launch of a new product with great potential, cannot be easily quantified by economists. For example, in September 2005, Hewlett–Packard spent approximately $600 million to acquire two data management software companies, Peregrine Systems and AppIQ.[1] The effect these acquisitions had on Hewlett–Packard's business may depend on a variety of qualitative factors, including how many key personnel remain in Peregrine and AppIQ after acquisition and how capably Hewlett–Packard integrates these firms' products into its overall marketing strategy. Presumably, HP analyzes the effects of

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[1] The Wall Street Journal, Sept. 20, 2005, page C4.

these business decisions both prior to and after its strategic move. However, its analysis is neither observable nor easily summarized by outsiders such as economists. Ignoring these unobservable productivity shocks creates biased estimation of production functions, as they affect the firm's choice of inputs.

A general form of a firm's production function takes the form: $y_{it} = f(x_{it}, \mu_{it}; \beta)$, where x_{it} is the set of inputs, μ_{it} is a productivity shock and β is a vector of parameters. To illustrate the analysis, we assume that the firm has a Cobb–Douglas production function with two major inputs, capital and labor. After taking the logarithm of both sides, the production function is:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \tag{1}$$

where y_{it} is log sales, k_{it} is log capital, and l_{it} is log labor. The unobserved part in the production function consists of two exogenous processes. ω_{it} is the Hicks neutral productivity shock, which is known to the firm at the time when variable inputs are selected. ϵ_{it} is a random error with zero mean in the production process, which is unpredictable by the firm. Unlike capital, the labor input is likely to be correlated with the Hicks neutral productivity shock. That is to say, firms can adjust their employment level contemporaneously when they observe the productivity shock. OLS gives biased and inconsistent estimates due to this simultaneity problem. Levinsohn and Petrin (2003) discuss the OLS bias using the same production function. They show that the actual bias can go either direction depending on how labor responds to the shock and how capital is correlated with labor. A natural alternative to OLS is to instrument labor with some variables which are correlated with the labor input but not with the productivity shock. However, for many applications of production economics, it is very difficult to find appropriate instruments, such as firm-level input prices.

Another approach is the fixed effect model. Firm fixed effects control for firm specific stable characteristics, and are appropriate for mature industries where there is little technological change. The estimating equation would be:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_i + \upsilon_{it} \tag{2}$$

where ω_i is a firm fixed effect, and υ_{it} is a random error. The biggest restriction the fixed effect model imposes is that ω_i has to be constant over time. For the computer industry, this restriction is inappropriate and the same problems of bias and inconsistency are still present.

A third method is to use a control function[2] as a proxy for unobservable productivity shocks. This method was introduced to the literature by Olley and Pakes (1996). It controls for unobserved productivity shocks using optimizing agents' decisions. These models permit firm characteristics to evolve over time. This greater generality comes at the expense of some complexity in estimation techniques. Olley and Pakes (1996) use decisions about capital investment and exit to control for the productivity shock, ω_{it}. They derive from their structural model strictly monotonic functions relating productivity, investment, and exit, and exploit these relationships to control for the productivity shock in a three stage estimation method. In their model, output is a function of a plant's marginal product ω_{it}, its age a_{it}, the level of capital stock k_{it}, labor l_{it}, and i.i.d. noise ϵ_{it}:

$$y_{it} = \beta_0 + \beta_a a_{it} + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \tag{3}$$

Capital is a state variable with law of motion $k_{it} = (1 - e^\delta)k_{it-1} + i_{it-1}$ where i_{it-1} is investment at time $t-1$ and δ is the rate of exponential depreciation of capital. The underlying economics is that the stock of capital is less flexible and needs time to adjust. To solve the simultaneity problem, Olley and Pakes (1996) show that the firm's investment i_{it} is a strictly increasing and therefore invertible function

of its current productivity shock, ω_{it}, any positive level of investment. So the function can be inverted to express the productivity shock:

$$\omega_{it} = f(i_{it}, k_{it}) \tag{4}$$

As a consequence, the productivity shock in the production equation can be replaced by Eq. (4) and the simultaneity problem is addressed. Olley and Pakes (1996) use both capital and a plant's probability of exit as proxies to estimate production functions. Levinsohn and Petrin (2003) show that intermediate inputs such as energy can also be used as a proxy for the productivity shocks. Ackerberg et al. (2006b) explore the critical nature of assumptions related to the timing of input decisions in these methods, and derive crucial principles for achieving identification. We show that, under certain circumstances, firm market value can also be used to obtain identification in production functions.

3. Theoretical model

We show that firm market value is a viable proxy for unobservable productivity shocks. A crucial property for firm market value to be a good proxy is that it must be strictly monotonic with respect to its productivity. Strict monotonicity means that the relationship between firm market value and the unobservable productivity shock can be 'inverted,' and this inverted function can be used to obtain identification. We prove strict monotonicity with a model of firm behavior based on Abel and Eberly (1994).[3] We derive an endogenous investment policy under convex and fixed adjustment costs. As is familiar in the literature, the firm adds to capital when its productivity crosses a certain boundary and liquidates itself when its productivity declines to a second lower boundary. As long as the firm is in operation, its market value is a strictly monotonic, and thus invertible, function of its productivity.

3.1. Description of technology

Suppose that a firm uses only labor and capital to produce a single output. The firm's operating profit function, ignoring the cost of capital for the moment, can be written as

$$\pi(K(t), L(t), \Omega(t)) = \Omega(t)^\alpha K(t)^\alpha L(t)^{1-\alpha} - P_l * L(t) \tag{5}$$

where $K(t)$ is capital, $L(t)$ is labor, P_l is the wage rate and $\Omega(t)$ is a Hicks neutral productivity shock which evolves continuously according to geometric Brownian motion

$$\frac{d\Omega}{\Omega} = \mu dt + \sigma d\xi \tag{6}$$

where μ is the drift and σ is the variance. Assume that the firm can only change its capital level at discrete annual intervals t_i, while labor is perfectly flexible. For every unit of capital investment, the firm has to pay two kinds of costs: the market price for the investment, and an adjustment cost. There are both fixed and quadratic costs components of the adjustment cost. The firm can buy capital for P_{kb} per unit, and can sell it for P_{ks}, where

$$P_{kb} > P_{ks} > = 0 \tag{7}$$

The price the firm receives for capital is lower that at which it can buy capital. This may be due to customization of capital for the firm's specific purposes or adverse selection in the resale market for capital.

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[2] See Ackerberg et al. (2006b) and Wooldridge (2005) for excellent reviews of this method.

[3] See Adda and Cooper (2003) and Caballero (1999) for reviews of the literature on dynamic approaches to investment.

In addition, suppose that the firm has both a fixed and a quadratic cost when it increases capital levels:

$$\Gamma(I) = X + \frac{\gamma I^2}{2} \tag{8}$$

Consequently, the total cost of investing in capital is $\Lambda(I) = P_{kb} * I + X + \frac{\gamma I^2}{2}$ if the firm invests in additional capital and zero if the firm does not adjust capital. The firm may sell all of its capital at once without any adjustment costs. Capital depreciates exponentially at rate δ.

3.2. Firm policy

At each instant, the firm chooses labor to maximize operating profits with the capital it has installed. It also periodically adjusts its capital level to maximize its market value. It chooses both when to invest and how much to invest, depending on its productivity level. Finally, if its productivity falls below a certain level, it will liquidate itself.[4] We derive its demand for labor and capital as well as its endogenous liquidation rule. We use these policies to determine the firm's market value.

Let ρ be the discount rate. At each instant, the firm seeks to maximize

$$\max_{I_t, L(t)} E_0 \int_0^\infty e^{-\rho t} \pi(K(t), L(t), \Omega(t)) dt - \sum_{j=0}^\infty \Lambda(I_j) \tag{9}$$

subject to the constraint that $K(\tau) = (K(t) + I(t))e^{-\delta(\tau - t)}$ for $t \leq \tau < t + 1$.

We first derive the demand for labor. As labor is instantaneously adjustable, profit maximization implies the firm's demand for labor, $L(t)$, is

$$L(t) = \left[\frac{(1-\alpha)}{p_L} \right]^{\frac{1}{\alpha}} \Omega(t) K(t) \tag{10}$$

which implies that operating profits are

$$\pi(K(t), \omega(t)) = P_L^{\frac{\alpha-1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} \Omega(t) K(t) \tag{11}$$

or $Z\Omega(t)K(t)$ where Z is a constant.

We now derive the demand for capital. Due to constant returns to scale in capital, the expected payoff to each unit if installed forever is

$$E_t \int_t^\infty e^{-(\rho + \delta)\tau} Z\Omega(\tau) d\tau = \frac{Z\Omega(t)}{(\rho + \delta - g)} \tag{12}$$

where $g = \mu + \frac{\sigma^2}{2} < \rho + \delta$.

If the economy is risk-neutral,[5] then the value of each unit of capital V_k must satisfy the partial differential equation:

$$\frac{1}{2}\sigma^2 \Omega^2 V_k'' + \alpha \Omega V_k' + Z\Omega - (\rho + \delta)V_k = 0 \tag{13}$$

where V_k' and V_k'' are first and second derivatives of the value function with respect to capital.

The firm can sell its capital for P_{ks}. Let Ω^* be the value of Ω at which the firm will liquidate itself. A solution to the value of capital will satisfy the value matching condition:

$$V_k(\Omega^*) = P_{ks} \tag{14}$$

and the smooth pasting condition:

$$V_k'(\Omega^*) = 0 \tag{15}$$

In Appendix A we show that the solution has the form:

$$V_k(\Omega) = A\Omega^\beta + \frac{Z\Omega(t)}{(\rho + \delta - g)} \tag{16}$$

where the first term represents the value of the option to sell capital, and the second term is the present value of profits from operations. A is a positive constant, and β is a negative constant. We also show that the firm will liquidate itself when $\Omega = \Omega^* = \frac{\beta}{\beta - 1} \frac{Z}{(\rho + \delta - g)}$.

Firm value is a function of its capital stock, which includes operating profits and the ability to sell the capital, plus its options to buy capital. We now turn to the value of these growth options. At each point in time where the firm can buy capital, the firm solves the static problem:

$$\max_{I_t \geq 0} \left(\left(A\Omega(t)^\beta + \frac{Z\Omega(t)}{\rho + \delta - g} \right) I_t - P_{kb} I_t - \frac{\gamma I_t^2}{2} - X, 0 \right) \tag{17}$$

The first term is the sum of the liquidation option and operating profits for each unit of capital. The other terms represent the cost of capital itself, plus quadratic and fixed adjustment costs.

The first order conditions from this maximization imply that the optimal level of investment is $\frac{1}{\gamma} \left(\frac{Z\Omega(t)}{\rho + \delta - g} + A\Omega(t)^\beta - P_{kb} \right)$ as long as $\frac{2-\gamma}{2\gamma^2} \left(A\Omega(t)^\beta + \frac{Z\Omega(t)}{\rho + \delta - g} - P_{kb} \right)^2 X$. Otherwise, the firm will not buy additional capital. The following summarizes firm investment policy:

Lemma 1. *Let Assumption 1 be that the firm's profit function be described as in Eqs. (5) and (6) and capital markets be as described by Eqs. (7) and (8). Assumption 1 is a sufficient condition that the firm's capital strategy can be characterized by three regions:*

Invest $\frac{1}{\gamma} \left(\frac{Z\Omega(t)}{\rho + \delta - g} + A\Omega(t)^\beta - p_{kb} \right)$ of additional capital if $\frac{2-\gamma}{2\gamma^2} \left(A\Omega(t)^\beta + \frac{Z\Omega(t)}{\rho + \delta - g} - P_{kb} \right)^2 > X$

Do not adjust capital if $\frac{2-\gamma}{2\gamma^2} \left(A\Omega(t)^\beta + \frac{Z\Omega(t)}{\rho + \delta - g} - P_{kb} \right)^2 \leq X$ and $\Omega > \Omega^*$.

Sell all capital if $\Omega = \Omega^*$.

3.3. Strict monotonicity of firm value

We now show that firm market value is strictly monotonic with respect to productivity, and thus is a candidate for use as a proxy when estimating production functions. The ability to add on capital at time τ is equivalent to a European power call option[6]:

$$C(\Omega, \tau, t) = \frac{(2-\gamma)e^{-\rho + \delta(\tau - t)}}{2\gamma^2} E_t \left[\left(\max \left\{ \left(A\Omega^\beta + \frac{Z\Omega(\tau)}{\rho + \delta - g} - P_{kb} \right)^2 - X, 0 \right\} \right) \right] \tag{18}$$

Firm value is the sum of the value of its capital stock and its options to expand,

$$V(\Omega(t), K(t)) = K(t) * V_k(t) + \Sigma_{\tau = t}^\infty (C(\Omega(t), \tau, t) \tag{19}$$

We show in Appendix A the following results:

Lemma 2. *Assumption 1 is a sufficient condition for $\frac{dC(\Omega(\tau), \tau, t)}{d\Omega(t)}$ 0 and $\frac{dV_k}{d\Omega(t)} > 0$.*

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[4] Since there are no adjustment costs for the sale of capital, the firm simply sells itself whole once its productivity falls below a certain endogenous level.

[5] In a risk neutral economy, investors and managers gain utility from a firms' expected profit, and ignore any covariance with a stochastic discount factor or other assets. We could assume some more complex asset pricing model but adopt risk neutrality for the sake of simplicity and tractability.

[6] It is 'European call' because it is the right to buy capital at a predetermined date. It is a 'power' option because its value upon exercise is a power of investment, rather than being linear in investment. See Nelkin (1996) for a discussion of power options.

The value of the capital from operations, with its embedded liquidation option, and the option to increase capital are both strictly monotonically increasing in productivity. Total firm value is simply the sum of the value of capital plus the growth options. This implies the following result:

Theorem 1. *Assumption 1 is a sufficient condition for the value of the firm to be strictly increasing in firm productivity $\Omega(t)$.*

Consequently, firm value, like investment, is a strictly monotonic and invertible function of Ω and can be used as a proxy for unobserved productivity shocks.[7]

4. Data

The empirical analysis is based on a data set we compiled from two sources: COMPUSTAT and NETS. COMPUSTAT provides firm level annual information on sales, capital stock, expenditures on purchased inputs and firm value.[8] The NETS database, a proprietary source based on Dun and Bradstreet establishment-level annual surveys, allows us to identify the Standard Industrial Classification (SIC) code and employment at each establishment owned by a firm. We restrict attention to computer related firms through matching firms' primary SIC code to the definition of the computer industry developed by Bardhan et al. (2003).

We use sales as our measure of output,[9] and the book value of plant, property, and equipment as a measure of capital stock.[10] We calculate research and development stock using the perpetual inventory method with a 15% depreciation rate.[11] Labor expense is calculated from establishment-level employment and SIC code information and state-level data on average compensation by SIC code. 'Other inputs' is the difference between 'Cost of Goods Sold' from Compustat and our estimate of labor expenditures. All inputs and outputs are in dollars, and are adjusted for inflation based on the producers' price index. Firm market value is defined as the sum of its stock and bonds.

Table 1 presents some summary statistics of the data. The data set is an unbalanced panel of 166 firms,[12] containing 1622 firm-year observations between 1989 and 2002. There are 106 entries over this period and 48 exits during this period. The total number of firms in computer industry varies from 81 in 1989 to 140 in 2002. We used business descriptions compiled by Hoovers[13] to identify the nature of the exit from our sample. The vast majority of firms exited due to being acquired by other firms or by going private. This brings the total number of firms in computer industry from 81 in 1989 to 140 in 2002. There is substantial heterogeneity in firm size; the value of sales varies from less than $13 million to over $75 billion. Some firms do not invest for particular years leaving only 1555 non-zero investment observations.

Table 1
Summary statistics.

	Obs.	Mean	Std. dev.	Min	Max
Sales	1622	3007.93	8448.49	12.06	75,780.31
Capital	1622	637.98	2096.42	0.12	26,078.94
Research	1622	1123.69	3457.23	0.14	32,737.16
Labor	1622	12.64	33.44	0.00	383.22
Other	1622	1569.77	4490.62	0.43	42,239.68
Market Value	1622	8472.54	32,142.55	2.92	596,475.80
Investment	1555	239.98	727.36	0.08	7309.00

Figures in the table are in millions of dollars.

5. Identification and estimation

5.1. Identification

We now discuss our estimation method and identifying assumptions. Our method integrates the use of firm market value as a proxy with the framework given in Ackerberg et al. (2006a). We assume that the production technology is Cobb–Douglas with four inputs: capital, research, labor, and other inputs such as materials and energy. Capital and research are fixed inputs. Labor and other inputs are variable inputs. We assume that all other factors affecting production known to the firm can be summarized by a single state variable. The main task for us is to estimate the coefficients of the logarithm transformed equation

$$y_{it} = \beta_k k_{it} + \beta_r r_{it} + \beta_l l_{it} + \beta_o o_{it} + \omega_{it} + \varepsilon_{it} \tag{20}$$

where y_{it} is log sales, k_{it} is log capital stock, r_{it} is log research stock, l_{it} is log labor input, and o_{it} is log other inputs for firm i and year t. There are two exogenous processes in the equation. ε_{it} is an i.i.d. error term which is not predictable. ω_{it} is the productivity shock which is known to the firm when labor and other inputs are chosen at time t and hence enters the firm's decision process. Note that the constant term in the production function is subsumed into the productivity term ω_{it}. Both ω_{it} and ε_{it} are not observed by the econometrician. Following the literature on estimation of production functions, we assume that capital (k_{it}) is set in the year prior to production. We also assume that research (r_{it}) is chosen by the firm at this point and becomes productive after a year. While a firm may be able to expand or contract resources devoted to research at a given point in time in response to productivity shocks, we expect the effects of such changes on productivity to occur with a substantial time lag.[14] Consequently, we assume that expenditures on research are set at $t-1$, and enter the production function at t. The timing of these decisions implies an orthogonality condition that we use to identify β_k and β_r.

We assume that labor and other inputs are freely adjustable. In contrast to capital and research, labor is non-dynamic and chosen at t.[15] As we discussed in Section 3, the optimal level of l_{it} is affected by the capital stock and the current productivity shock, ω_{it}. Similarly, the choice of other inputs should be affected by the same set of factors because some components of other inputs are even more flexible than labor. Explicitly, we may think of two possible data generating processes (DGP) for labor

$$l_{it} = \psi_t(\omega_{it}, k_{it}, r_{it}) \tag{21}$$

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[7] It may be possible to extend our strict monotonicity result to much more general dynamic models under the assumption that the distribution $P(\Omega(t+1)|\Omega(t))$ is stochastically increasing in $\Omega(t)$. Under this assumption, a higher $\Omega(t)$ not only leads to strictly higher current profits, but also higher future profits. Hence, under appropriate conditions, the value function should be strictly increasing in $\Omega(t)$.

[8] Wallace and Walls (2004) has detailed discussion of construction of the data set.

[9] We use sales rather than physical output due to data limitations. Klette and Griliches (1996) discuss the biases that result from the use of sales rather than quantity in production functions estimated by OLS.

[10] Using the perpetual inventory method for capital would require deleting two-thirds of our sample due to missing data on investment. The correlation between the book value of capital and capital stock calculated from the perpetual inventory method for the sub-sample where this was possible is 0.9588.

[11] We chose a depreciation rate of 15% based on Hall and Mairesse (1995).

[12] There are some interruptions in some firms' series due to mergers, acquisition, spin-offs, and changes from public to private ownership. We treat the same firm before and after the interruption as different firms.

[13] Hoovers is a subsidiary of Dun and Bradstreet, see www.Hoovers.com.

[14] As Griliches (1979) states, "A particular research and development project may take more than a year to complete. Second, when complete and if successful, it may still take some time before a decision is made to use it, or produce it. Once an innovation decision is made, it may show up in the firm's revenue stream only with another lag."

[15] Bond and Soderbom (2005) show that in Cobb–Douglas production functions, parameters on perfectly variable and non-dynamic inputs are not identified without input price variation. The firms within our sample have a wide variety of locations for factories and research labs, and thus are likely to experience localized i.i.d. firm-specific shocks to the prices of labor and other inputs. These i.i.d. price shocks permit estimation of the production function.

and for other inputs

$$o_{it} = \varphi_t(\omega_{it}, k_{it}, r_{it}) \tag{22}$$

That said, the choices of variable inputs at t depend on predetermined values of two fixed inputs and current productivity shock. As a consequence, variable inputs are potentially correlated with ω_{it}, causing endogeneity, and with fixed inputs, causing collinearity.

To identify the Eq. (1), we need to address both endogeneity and collinearity. Our identification strategy follows that of Ackerberg et al. (2006b). Firm market value, m_{it}, which we define as the sum of the book value of debt and firm market value of equity, reflects information used by the firm when it chooses its variable inputs. Investors receive regular updates from firms through conference calls with analysts, press releases, and other means. An active legal system provides firms with an incentive to release information in a prompt manner. In particular, the solution to the firm's optimization problem results in firm market value equation

$$m_{it} = \phi_t(\omega_{it}, k_{it}, r_{it}) \tag{23}$$

where ϕ_t is allowed to vary by year, representing changing macroeconomic conditions. We have shown that firm market value, m_{it}, is strictly monotonic in the production shock, ω_{it}. So for every set of (k_{it}, r_{it}), we can invert Eq. (23) to get an expression for ω_{it}:

$$\omega_{it} = \phi_t^{-1}(m_{it}, k_{it}, r_{it}) \tag{24}$$

Thus, the unobserved productivity shock is controlled by the observables. Substituting Eq. (24) into Eq. (1), we can rewrite the production function as

$$y_{it} = \beta_k k_{it} + \beta_r r_{it} + \beta_l l_{it} + \beta_o o_{it} + \phi_t^{-1}(m_{it}, k_{it}, r_{it}) + \varepsilon_{it} \tag{25}$$

Notice that none of the parameters is identifiable due to collinearity between fixed inputs (l_{it}, o_{it}) and variable inputs (k_{it}, r_{it}).

We do not try to identify parameters of labor and other inputs in the first stage. Instead, we focus on estimating the conditional moment, $\hat{\Phi}_{it}$, using flexible non-parametric smoothing

$$\hat{\Phi}_{it} = E(y_{it}|m_{it}, k_{it}, r_{it}, l_{it}, o_{it}) = \beta_k k_{it} + \beta_r r_{it} + \beta_l l_{it} + \beta_o o_{it} + \phi_t^{-1}(m_{it}, k_{it}, r_{it}) \tag{26}$$

which is the expected output given all inputs and firm market value. It is a strategy to separate the productivity shock from the unanticipated error in the production process. Then, given a set of parameters β, the productivity shock, ω_{it}, can be recovered as

$$\omega_{it} = \phi_t^{-1}(m_{it}, k_{it}, r_{it}) = \hat{\Phi}_{it} - \beta_k k_{it} - \beta_r r_{it} - \beta_l l_{it} - \beta_o o_{it} \tag{27}$$

We need at least four independent moment conditions to identify the parameters in the second stage. Assuming that the productivity shock follows a first order Markov process, we have an expression for the innovation in productivity at t

$$\xi_{it} = \omega_{it} - E(\omega_{it}|\omega_{it-1}) \tag{28}$$

It can be estimated as the residual of the OLS regression with ω_{it} as the dependent variable and a higher order polynomial of ω_{it-1} as regressors.[16] Based on our set up, ξ_{it} should be mean independent of

Table 2
Estimates using firm market value proxy and firm fixed effects.

	Value proxy	Firm fixed effect	Differences
	(1)	(2)	(3)
Capital	0.2236** (0.0582)	0.2387** (0.0453)	− 0.0151 (0.0567)
Research	0.0607** (0.0243)	0.0499** (0.0156)	0.0108 (0.0255)
Labor	0.1084 (0.1326)	0.1859** (0.0607)	− 0.0775 (0.1151)
Other	0.5348** (0.0857)	0.4877** (0.0664)	0.0471 (0.0775)

The first column presents estimates of the production function with the firm market value proxy. The second column presents estimates of the production function using firm fixed effects and OLS. The third column shows the difference between two sets of estimates. The data set includes 1622 observations and 166 firms. However, some firms have a discontinuity in the time series for investment in capital goods. We treat the same firm before and after the discontinuity as different firms. Finally, we end up with 190 firms.
** and * indicate p-value smaller than 0.01 and 0.05. Standard errors (in parentheses) are robust to clustering or bootstrapped with firm-level autocorrelation. The bootstrapped standard errors are based on 200 replications.

two state variables, k_{it} and r_{it}, because they are decided at $t-1$. For labor and other inputs, ξ_{it} is generally correlated with l_{it} and o_{it} since we allow them to adjust to information available at t. However, lagged labor and other inputs are chosen at $t-1$. This implies that ξ_{it} should be independent of l_{it-1} and o_{it-1}. Finally, we obtain four independent moment conditions

$$E\left(\xi_{it} * \begin{pmatrix} k_{it} \\ r_{it} \\ l_{it-1} \\ o_{it-1} \end{pmatrix}\right) = 0 \tag{29}$$

for the identification of four parameters.[17] Since the model is just identified, the Method of Moment estimators solve the following objective function:

$$Q(\beta^*) = \min_\beta \sum_{i=1}^N \sum_{t=2}^T (\xi_{it}(\beta) * k_{it})^2 + (\xi_{it}(\beta) * r_{it})^2 \\ + (\xi_{it}(\beta) * l_{it-1})^2 + (\xi_{it}(\beta) * o_{it-1})^2 \tag{30}$$

5.2. Results

We first estimate the model using the full data set with 1622 observations. Some firms have missing periods in the time series. For example, a firm may be listed from 1989 to 1995, go private or be acquired, and then go public again or be spun off and be listed 1998 to 2002. We treat the same firm where there are such gaps as different firms in the analysis. The results are presented in Table 2. Standard errors are calculated using bootstrapping with 200 replications.

Both methods produce similar coefficients on the fixed inputs, capital and research; capital is slightly higher, while research and development is slightly lower using the value proxy. The coefficient on labor is almost cut in half with the value proxy, while that on other inputs increases by about 10%. None of the differences between the coefficients are statistically significant.

In order to compare our results with prior work we also estimate the production function using an investment proxy. The sample size is reduced to 1555 due to missing values in investment. We estimate the C–D production function using both proxies on the each sample. The results are presented in Table 3.

There are some interesting differences between two estimators. The value proxy model produces bigger capital coefficients and smaller other inputs coefficients than the investment proxy model for every

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[16] Local linear regression will also serve this purpose, but it would greatly increase the computing time because we have to calculate ξ_{it} for every converge attempt.

[17] Alternatively, we may use more moment conditions and estimate the parameters using GMM. For example, ξ_{it} should also be mean independent of lagged capital and research. This gives us two more moment conditions: $E(\xi_{it} * k_{it-1}) = 0$ and $E(\xi_{it} * r_{it-1}) = 0$.

Table 3
Estimates using firm market value proxy, investment proxy and firm fixed effects.

	Value proxy	Investment proxy	Firm fixed effects	Differences between (1) and (3)	Differences between (1) and (2)
	(1)	(2)	(3)	(4)	(5)
Capital	0.2116** (0.0542)	0.1968** (0.0569)	0.2264** (0.0479)	− 0.0148 (0.0671)	0.0148 (0.0129)
Research	0.0447* (0.0228)	0.1006** (0.0254)	0.0468** (0.0156)	− 0.0021 (0.0269)	− 0.0559** (0.0168)
Labor	0.1039 (0.1303)	0.1154 (0.1430)	0.187** (0.061)	− 0.0831 (0.1224)	− 0.0115 (0.0262)
Other	0.5577** (0.088)	0.5372** (0.0898)	0.5123** (0.0735)	0.0454 (0.0834)	0.0205 (0.0167)

The first column presents the estimates of the production function using the firm market value proxy. The second column presents the estimates of the production function using the investment proxy. The third column presents the estimates of the production function using firm fixed effects and OLS. The fourth column presents the differences between the value proxy model and the firm fixed effects model. The last column presents the differences between the value proxy model and the investment proxy model.
** and * indicate p-value smaller than 0.01 and 0.05. Standard errors (in parentheses) are robust to clustering or bootstrapped with firm-level autocorrelation. The bootstrapped standard errors are based on 200 replications.

bootstrap replication. *p*-values are zero for the tests that capital (other inputs) coefficient of the value proxy model is greater (smaller) than that of the investment proxy model. For the research coefficient, the investment proxy estimate is bigger than the value proxy estimate. The difference is big relative to the magnitude of the coefficients, but there are still 12.5% of bootstrap repetitions in which the value proxy estimates are bigger than the investment proxy estimates. This suggests that our value proxy performs similarly to the investment proxy because the coefficients are statistically indistinguishable from each other. Compared with estimates with full sample, the value proxy model produces remarkably similar results with the censored sample. While the differences are not statistically significant, the fixed effect model does seem to overstate the marginal elasticity of sales with respect to labor. The primary lesson we take from this exercise is that the value proxy generates similar results to the investment proxy when applied to our sample of computer industry firms.

6. Conclusion

Production functions, which relate firms' inputs to their outputs, are a crucial tool for understanding technology. Unobserved productivity shocks, such as managerial quality and strategic decisions, affect the choice of inputs. It is often hard to objectively measure these shocks, creating a formidable endogeneity problem. Olley and Pakes (1996) develop an estimator which allows for time-varying firm quality. They rely on investment being monotonic in the marginal product of capital. We show that firm value shares this crucial property, and thus offers another potential proxy for firm heterogeneity. We test the value proxy on a sample of firms from the US computer industry.

There are several limits to our work. Our model assumes that the production function is linearly homogenous and Cobb–Douglas in functional form. Our method is limited to publicly traded firms. The fact that we use sales, rather than a physical measure of output and that the firms we study typically produce more than one kind of product may bias our results.[18] Future work will explore the applicability of this work to multi-product firms.

Appendix A

Derivation of the value of capital V_k and liquidation boundary Ω^*

A general solution to the partial differential equation has the form

$$V_k = A_1\Omega^{\beta_1} + A_2\Omega^{\beta_2} + Z\frac{\Omega(\tau)}{\rho + \delta - g} \tag{31}$$

where A_1, A_2, β_1, and β_2 are constants, $\beta_1 > 0$ and $\beta_1 < 0$. As Ω grows without bound, it becomes very unlikely that the firm would ever sell its

capital, so the value of the capital should approach $Z\frac{\Omega(\tau)}{\rho + \delta - g}$. This implies that $A_1 = 0$ and we can write the value of each unit of capital as:

$$V_k = A\Omega^{\beta} + Z\frac{\Omega(\tau)}{\rho + \delta - g} \tag{32}$$

We can write the value matching condition as:

$$A\Omega^{\beta} + Z\frac{\Omega(\tau)}{\rho + \delta - g} = P_{ks} \tag{33}$$

while the smooth pasting condition is

$$\beta A\Omega^{\beta-1} + \frac{Z}{\rho + \delta - g} = 0 \tag{34}$$

Some algebra implies solutions for the unknown constants:

$$A = \frac{P_{ks}}{(1-\beta)\Omega^{*\beta}} \tag{35}$$

$$\Omega^* = \frac{\beta(\rho + \delta - g)P_{ks}}{Z\beta - 1} \tag{36}$$

Proof of Lemma 1

1) Show that $\frac{dV_k}{d\Omega(t)}$ 0.
   By the smooth pasting condition, the first derivative of V_k with respect to Ω is zero when evaluated at Ω^*. The second derivative of V_k is

$$\frac{d^2V_k}{d\Omega^2} = (\beta-1)\beta A\Omega^{\beta-2} \tag{37}$$

   which implies that V_k is strictly convex with respect to Ω since $\beta < 0$ and both A and Ω are positive. Consequently, for all values of $\Omega > \Omega^*$ the first derivative is positive.

2) Show that $\frac{dC(\Omega(t),\tau,t)}{d\Omega(t)} > 0$
   a) Show there exists a Ω^{**} such that

$$\frac{2-\gamma^2}{2\gamma}\left(A\Omega^{\beta} + Z\frac{\Omega(\tau)}{\rho + \delta - g} - P_{kb}\right)^2\right) = X \tag{38}$$

   Since Ω is a geometric Brownian motion, it is strictly positive, so it is sufficient to show the existence of a unique Ω^{**} such that

$$A\Omega^{\beta} + Z\frac{\Omega(\tau)}{\rho + \delta - g}) = P_{kb} + \sqrt{\frac{2-\gamma^2}{2\gamma}X} \tag{39}$$

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[18] See earlier versions of this paper, available upon request from the authors, for some exploration of these issues.

We showed in the first section of the appendix that the left hand side of this equation is strictly increasing in $\Omega > \Omega_{ks}$. By the smooth pasting condition, it is also equal to P at Ω^*, the point at which the firm liquidates itself. As Ω increases without bound, so does the left hand side. Consequently, by the Mean Value Theorem, Ω^{**} exists.

b) The value of the call is

$$
\begin{aligned}
C(\Omega, \tau, t) &= \frac{(2-\gamma)e^{-\rho + \delta(\tau - t)}}{2\gamma^2} E_t\left[\max\left(\left(A\Omega(\tau)^{\beta} + \frac{Z\Omega(\tau)}{\rho + \delta - g} - P_{kb}\right)^2 - X, 0\right)\right] \\
&= e^{(-\rho + \delta)(\tau - t)} \frac{2-\gamma^2}{2\gamma} \int_{\zeta^{**}}^{\infty}\left[\left(A(\Omega(t)\zeta)^{\beta} + Z\frac{\Omega(t)\zeta}{\rho + \delta - g} - P_{kb}\right)^2 - X\right]f(\zeta)d\zeta
\end{aligned}
$$
(40)

where $\zeta^{**} = \frac{\Omega^{**}}{\Omega(t)}$.

By Leibniz's Theorem,

$$
\begin{aligned}
\frac{dC(\Omega, \tau, t)}{d(\Omega(t))} &= e^{-(\rho + \delta)(\tau - t)} \frac{2-\gamma^2}{2\gamma} \int_{\zeta^{**}}^{\infty}\left(A(\Omega(t)\zeta)^{\beta} + \frac{Z\Omega(t)\zeta}{\rho + \delta - g} - P_{kb}\right) \\
&\quad \times \left(A\beta\Omega(t)^{\beta-1}\zeta^{\beta} + \frac{Z\zeta}{\rho + \delta - g}\right)f(\zeta)d\zeta
\end{aligned}
$$
(41)

The first term after the integral is

$$
A\Omega(\tau)^{\beta} + \frac{Z\Omega(\tau)}{\rho + \delta - g} - P_{kb}
$$
(42)

The second term when multiplied by $\frac{\Omega(t)}{\Omega(t)}$ is

$$
\left(A\beta\Omega(\tau)^{\beta} + \frac{Z\Omega(\tau)}{\rho + \delta - g}\right)\frac{1}{\Omega(\tau)}
$$
(43)

The expression in Eq. (42) is the marginal effect of an additional unit of capital on firm value. It strictly positive for all $\Omega(\tau) > \Omega^{**}$ the region where the firm invests. The expression in Eq. (43) is the same as Eq. (42), divided by a geometric Brownian motion, which is always positive. $f(\zeta)$ is also always positive since it is a probability density function. The leading terms are also strictly positive. Consequently, $\frac{dC(\Omega(t), \tau, t)}{d\Omega(t)} > 0$.

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